Some Experiments in Constructing a Hybrid Model For Macroeconomic Analysis*

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Abstract

VAR analysis is a widespread method of quantitatively analyzing macro-economic issues. In this paper we examine the use of "hybrid" VAR models that retain the short-run features of a VAR but are designed to reproduce selected characteristics of calibrated models that are frequently used for the simulation of policy actions. The calibrated model we use is the McKibbin Sachs Global (MSG2) model of the world economy. For permanent shocks we constrain the long-run responses in the hybrid model to match those from MSG2. For transitory shocks we match shorter-run cumulative responses. The estimated effects of a permanent US money supply shock are broadly consistent with those of MSG2, but differ in some dimensions from those obtained from a standard recursive VAR.

Introduction

The determination of the impact of policy actions upon an economy has long been a central research area in macroeconometrics. In the beginning it looked to be an easy task. One would just construct large-scale models of the economy to emulate the IS-LM framework, differing mainly in the degree of disaggregation, and then simulate these models to gauge the impact of a change in a policy variable, such as the money stock. This early optimism faded as a realization of some of the difficulties of large-scale models became apparent. Some of these reflected a weak theoretical base - no supply side constraints, poorly defined expectations formation, weak interaction of stocks and flows, and a failure of many models to converge to a satisfactory equilibrium position once a variable is changed.

Modelers learned from these difficulties and set about rectifying them in various ways. In particular, models have appeared that feature rational expectations, steady state growth paths, and stocks influencing flows. There is considerable diversity in their nature but those that are to be used for policy analysis tend to work with the sticky price paradigm in the short run and exhibit classical properties in the long run. These “theory models” have become very popular as a way of thinking about the impact of policies, as the responses revealed by them can be rationalized by their nature. However, their emphasis tends to be on responses in the medium rather than the very short run and the unit of time they work with is sometimes lengthier than desirable e.g. at a yearly interval rather than a monthly one, and this can be a limitation.

As an example of such a model consider the McKibbin Sachs Global (MSG2) multi-county model. This is a dynamic intertemporal general equilibrium model of the world economy.[1] The version used here consists of country models for the United States, Japan, Germany and Australia along with aggregated models for other groupings, such as the remainder of the EMS and the OECD, high income Asia, other Asia, Eastern Europe and the former Soviet Union, and oil exporting developing countries. Its parameters have been calibrated using information from each country.

The model has a number of attractive features for policy analysis. First, the long run of the world economy is well determined, being driven by a Solow-Swan-Ramsey neoclassical growth model, with exogenous technical progress and population growth in different economies. In the short run, however, the dynamics of the global economy towards this growth path are determined by a number of Keynesian style rigidities in the goods and labor markets. This is important for the tracking performance of the model as well as capturing the dynamic adjustment path towards long run global equilibrium. Households and firms are assumed to maximize intertemporal utility and profit functions subject to intertemporal budget constraints. In the short run some proportion of firms and households are assumed to use optimal rules of thumb rather than recalculating the entire intertemporal equilibrium of the model (alternatively these could be interpreted as liquidity constraints). Wages are assumed to adjust slowly to clear labor markets subject to the institutional characteristics of labor markets in the different economies.

Most importantly, stock-flow relations are imposed in the model. Private investment leads to physical capital accumulation, public investment spending leads to the accumulation of a stock of infrastructure capital, fiscal deficits lead to accumulation of government debt and current account deficits lead to an accumulation of foreign claims against domestic production. Intertemporal budget constraints are imposed so that all outstanding stocks of assets must be ultimately serviced. Another important feature of the model is that asset markets are efficient, in the sense that asset prices are determined by a combination of intertemporal arbitrage conditions and rational expectations. Asset prices are directly tied down by the imposition of intertemporal budget constraints in the model. The long run behavior of the model depends on stock equilibrium; asset prices stabilize in real terms, once the desired ratios of asset stocks to GDP are reached.

The MSG2 model can be used in a number of ways to suggest what the likely effects of policy changes in the U.S. would be. There are two basic scenarios that might be used. In the first, an unanticipated transitory change is made to some policy or exogenous variable lasting a single period. In the second, an unanticipated (for the first period) permanent change is made. Figures 1-3 show a use of the model, giving the results of an unanticipated transitory 1% change in “the U.S. money supply” upon a variety of indicators over a period of 10 years, *inter alia*.

A second theme in macroeconomic research, which also originated from doubts about large-scale models, was the growth of VAR’s. The underlying concern responsible for this development was the fact that the identification problem within a system of equations was being solved via restrictions upon the dynamics, leading to the proposal that there be either none or few dynamic restrictions placed upon the system. These models also introduced a different way of thinking about policy experiments. Here monetary actions were regarded as "shocks", emphasizing that they were unpredictable events from the viewpoint of the information within the VAR. In the early Cowles Commission tradition, the fact that there were errors in the equations was largely ignored, and these were certainly not thought of as policy variables, although the practice of performing policy experiments by changing the intercepts in equations could be interpreted as involving shocks. As an example of a result from a VAR based model, Figures 1-3 also displays the effects of an unanticipated transitory 1% increase in the quantity of non-borrowed reserves upon selected variables from a monthly VAR model referred to as the CP model in Pagan and Robertson (1995). It consists of six equations for the log of the industrial production index, the log of the CPI, the log of an index of commodity prices, the log of non-borrowed reserves, the Federal Funds rate and the log of total reserves. The system is recursive in the order described, includes six lags, a constant and a trend term, and estimation is over the period 1959M1 to 1994M1.[2] Apart perhaps for the initial negative effects on output the responses are standard for this type of model, with prices and output rising for some time, and short-run interest rates temporarily declining.

It is interesting to dwell on the differences between the predictions of the effects of an unanticipated monetary shock made by the CP and MSG2 models, as revealed in Figures 1-3. A comparison is not entirely straightforward, principally because the experiment conducted on the MSG2 model pertains to a yearly rather than monthly interval of time. This forces us to give some interpretation to the MSG2 results. To explain what we do, take the raw result from an MSG2 simulation that a 1% unanticipated yearly rise in the money stock produces a decline in the nominal short run interest rate of 68 basis points. We interpret the latter as being a decline in the average rate of interest in the year after the experiment is performed relative to the base period year. In the same way the money stock is taken to have increased on average by 1%. Because this is an average over a year, reproducing a shock that persists for only a single month requires a 12% shock for the first month. Consequently, we conclude that MSG2 predicts that an unanticipated 1% increase in the money supply for a single month will lead to a decline in the short run interest rate of 68/12 = 5.7 basis points, the number recorded in Figure 3.

Figures 1 to 3 about here

Given this strategy one would like a succinct summary of the differences in the figures. The traditional focus of many VAR studies is upon the sign and shape of impulse responses, but this seems to be too diffuse. A different perspective is to be had by cumulating the impulse responses of the two models and then comparing the difference between them at some horizon. Formally, one is comparing the area under the curves. In some instances the comparison may also be meaningful, e.g. in the case of output it provides the aggregate of all the output changes from the monetary injection. These cumulative impulse responses effectively address a different dimension of impulse responses than the traditional ones, namely their *magnitude* rather than just their *sign and shape*. For example, the CP model predicts long-run cumulative effects on output, the price level and the federal funds rate of 0.93%, –0.27%, and 29 basis points for a shock that leads to a cumulative 1% rise in non-borrowed reserves. A comparable shock in the MSG2 model would set these cumulative responses at 0.19%, 0.44% and −64 basis points respectively.[3] Viewed in this way the differences between the effects of a monetary change are quite large and some of the results, such as the initial negative output response in the CP model, are hard to rationalize.

As the example illustrates, starting from a common perception of the need to modify modeling practice, these two approaches have now diverged by emphasizing separate deficiencies in the original system-wide methods of analyzing policy actions. It would seem unfortunate that this division has arisen. The theory models are attractive since they tell a story about monetary policy that can be understood from economic reasoning, while the VAR approach appeals because it responds to what is in the data and provides much finer detail regarding the dynamic responses. Hoover et al (1997, p. 41) summarize this division in an appealing way, albeit in a different context:

> “ The dilemma is this: theories are interpretable, but too simple to match the features of the data; rich econometric specifications are able to fit the data, but cannot be interpreted easily”.

How should one respond to this dilemma? There seem to be two broad strategies for marrying the two traditions. One is to develop the theory models so as to produce complex dynamics. This is not an easy task when the observation period is a month or a quarter. In practice, stochastic dynamic general equilibrium models have not fared well when called upon to accurately model short term dynamics and it is rare to find models in this class that will match the data on many dimensions. Perhaps this will not be so one day. The second is to explore the possibility of using the information provided by simulations of models such as MSG2 in order to modify the VAR. We will term such models hybrid as they aim to capture some of the good characteristics of a VAR, such as its ability to represent short term-dynamics, but also to retain some of the information available from a model that one might use for policy work.

In the next section we formally set out some strategies for constructing a hybrid model. There is no one solution to this task and in practice one might expect a range of options to be canvassed. In section 3 of the paper we look at the issue of measuring the effects of monetary actions by suitably constraining a fairly standard VAR model. As the example illustrates there are many difficult issues in producing hybrid models and, although we have learned a lot from attempting this, a good deal of work remains to be done.

Modeling Strategies

This section considers some ways in which hybrid models might be constructed. There are two cases to be considered. In the first all shocks are viewed as being transitory. A lot of research with VAR's seems to have this philosophy as its foundation. In the second, shocks are regarded as either being strictly permanent or there is a combination of transitory and permanent shocks. The methods needed to formulate a hybrid model vary according to each case and so will be discussed separately.

Pure Transitory Shocks

Consider a stationary *p*'th order VAR in *n* variables y(t),

$$y(t) = A_1 y(t-1) + ... + A_p y(t-p) + e(t) \quad (1)$$

or

$$A(L)y(t) = e(t), \quad (2)$$

where

$$V = cov(e(t)). \quad (3)$$

Derivable from this is the MA representation describing the impulse responses:

$$y(t) = (I+G_1L+\ldots)e(t) = G(L)e(t). \quad (4)$$

It is possible to estimate A(L) and to then compute an estimate of G(L). The quantities G(L) and V can then be taken as summarizing the data. The corresponding representation of the hybrid model is

$$y(t) = (C_0 + C_1L+\ldots)\varepsilon(t) = C(L)\varepsilon(t). \quad (5)$$

The hybrid model is one that uses some of the theoretical framework of MSG2, as represented by certain constraints on the nature of C(L), and which is statistically indistinguishable from (2). The key to implementing the idea is to regard the hybrid model shocks, $\varepsilon(t)$, as being a non-singular transformation of the VAR errors i.e. $\varepsilon(t) = T^{-1}e(t)$, so that

$$C(L) = G(L)T. \qquad (6)$$

Provided T is non-singular the constraint that the hybrid model must agree exactly with the VAR in (2) is thereby enforced - exact agreement meaning that the hybrid model can be precisely transformed to (2). Using standard terminology, the hybrid model can be thought of as a "just identified VAR", meaning that it would be impossible to distinguish between it and any other model which also agreed exactly with (2).

The question then becomes one of how to estimate T. One simple solution is to use the total cumulative responses to both types of shocks, i.e. to find T such that C(1)=G(1)T, where C(1) is taken from the MSG2 simulations and G(1) is estimable from the data. This type of restriction results in the matching of a particular characteristic of the impulse responses, C(1), while other aspects of C(L) are free to be determined by the data. In fact, the "estimated" impulse responses of an unanticipated policy shock in the hybrid model will now be G(L)T since we would have $y(t) = G(L)TT^{-1}e(t)$. Notice that C(1) is not the only characteristic that might be emulated. Defining the matrices attached to L^j in (6) as C_j and G_j it is also true that $C_j = G_jT$ and $\Sigma_{j=1,M}C_j = (\Sigma_{j=1,M}G_j\,)T$, allowing one to solve for T by using the cumulated sum of the impulse responses up to any pre-defined number (M). One might wish to do this in the light of contentions in Faust and Leeper (1997) relating to the potential difficulty of estimating G(1).

Two comments might be made about the approach. The first relates to the fact that we are making no reference to $\varepsilon(t)$ as "structural shocks". In most investigations of this sort T would be the inverse of the contemporaneous matrix linking structural and reduced form errors. Here the best way to think of a shock is via the following aphorism: "a shock is what a shock is supposed to do". For example, a money shock will be that combination of VAR errors which reproduces a set of designated characteristics of the responses to a money shock as given by the MSG2 simulations. Clearly, what a "money shock" is will vary with the theory model used, as well as with the characteristics of the data that are to be replicated. A second observation is that the shocks in $\varepsilon(t)$ may have a non-zero correlation. Indeed, since $cov(\varepsilon(t)) = T^{-1}V(T')^{-1}$, it is likely that the hybrid model's shocks will be correlated if the data, as represented by (2), is to be replicated.

In some instances it may be that the number of restrictions we wish to use from the theory model is less than the number of elements of T. For example, suppose we partition C(1) as $C(1) = [C_{MSG}(1) \;\; C_{NMSG}(1)]$, where only $C_{MSG}(1)$ is assumed known from MSG2. When C(1) is completely known T can be recovered exactly, showing that the issue becomes one of obtaining information on quantities such as $C_{NMSG}(1)$ in order to complete the analysis. For this we need some other restrictions. If only information from the VAR in (1) is to be used then the remaining item that is available to be exploited for this purpose is the covariance matrix of e(t), V. To be more precise, write $T = [T_{MSG} \;\; T_{NMSG}]$ so that

$$T = G(1)^{-1}C(1) = G^{-1}(1)[C_{MSG}(1) \;\; C_{NMSG}(1)]. \qquad (7)$$

Equation (7) shows that $T_{MSG} = G(1)^{-1}C_{MSG}(1)$, leaving T_{NMSG} to be determined. Because $V=Tcov(\varepsilon(t))T'$ it is clear that some assumption relating to $cov(\varepsilon(t))$ might enable one to estimate T_{NMSG}. To give an example that relates to our later empirical work suppose that n=6 and three columns of $\Sigma_{j=1,M}C_j$ are assumed known from MSG2. This results in 18 unknown elements in T_{NMSG}. Three of these may be set to unity as a normalization and so fifteen unknowns remain. Making the assumption that the shocks $\varepsilon(t)$ are uncorrelated produces a total of 21 unknown elements in the hybrid model - 15 in T_{NMSG} and the six unknown variances. To determine these one has 21 known elements in V. Of course, one is simply counting unknowns and equations and there is no certainty that one can find estimates that obey the restrictions e.g. it must be the case that the estimated variance estimates are non-negative.[4]

Combined Permanent and Transitory Shocks

As mentioned in the introduction other simulations can be performed with a model like MSG2. In particular, permanent shocks may be applied. In MSG2 such a shock is also easier to interpret than a transitory one, as no decision needs to be made about how to distribute it across a year when working with monthly data. To exploit this information leads one into the realm of integrated and cointegrated processes. Suppose the *n* variables in x(t) are I(1) processes. Then the assumption that there are *k* permanent shocks implies that there are $r = n-k$ cointegrating relations amongst the elements of x(t) and the VAR in (2) would become a VECM of the form

$$A^{*}(L)\Delta x(t) = \alpha\beta' x(t-1) + e(t). \tag{8}$$

In equation (8) β is the $n\times r$ matrix of cointegrating vectors, α is the $n\times r$ matrix of loadings and $A^{*}(L)$ is a $(p-1)$'th order polynomial. The MA representation connecting the stationary $y(t) = \Delta x(t)$ with the VAR errors e(t) has the form

$$\Delta x(t) = D(L)e(t), \tag{9}$$

while the MA for the levels x(t) is

$$x(t) = \Phi(L)e(t), \tag{10}$$

where $\Phi(L) = (1-L)^{-1}D(L)$. In the same way the hybrid model MA representations will be

$$\Delta x(t)= C(L)\varepsilon(t), \tag{11}$$

and

$$x(t) = \Psi(L)\varepsilon(t) \tag{12}$$

with $C(1) = \Psi_{\infty}$ showing the eventual impact of the hybrid model's shocks upon the "long-run" level of x(t). Here, we take C(1) to be given by the long-run responses to *k* permanent shocks in MSG2 as well as to $r \geq 0$ transitory shocks which may or may not be identified with MSG2.[5]

By and large the literature interested in using information about the long run effect of permanent shocks has asserted that there are *n* permanent shocks, in which case there is no cointegration ($r = 0$) and (8) becomes

$$A^{*}(L)\Delta x(t)=e(t). \quad (13)$$

Much of this literature also imposes restrictions upon the rows of C(1), as distinct from the columns, and the information on them comes from some generalized theoretical perspective e.g. Gali (1992) has money demand, money supply and an IS shock having no long run effect upon output; only the "diagonal term", taken to be a supply side shock, has a non-zero effect. A row restriction describes the effects of *all* shocks in the system upon a particular variable in the steady state. In contrast, a column restriction describes the steady state effects on all the variables in the system of a *specific* shock. A problem with row restrictions is that they are very demanding. It seems unlikely that one can specify a row of C(1) in many realistic contexts since the number of shocks becomes quite large, making it hard to interpret them. For example, in Leeper et al's (1996) large scale VAR's, shocks are just given generic names such as "private sector", and it would be difficult to ascertain what a suitable restriction upon a row of C(1) would be.

Restrictions on the columns of C(1) have also been used in the literature. Lastrapes and Selgin (1995) use one column restriction when maintaining that money shocks have no effect upon output, the nominal interest rate and real money balances in their four variable system. Of course, a single column restriction would not suffice for identification of all the permanent shocks without some extra information. In their case identification is achieved by making C(1) lower triangular along with the assumption that the $\varepsilon(t)$ are uncorrelated. Unfortunately, lower triangularity represents a set of restrictions whose appeal is in inverse proportion to the order of the model; as the model expands in size it becomes increasingly hard to interpret all the permanent shocks. Most applications of this idea have been restricted to a small number of variables. For this reason the ability of MSG2 to provide column restrictions is interesting in terms of its capacity to handle large systems of variables.

If there is no cointegration among the x(t) a hybrid model incorporating MSG2 outcomes for permanent shocks is constructed in exactly the same way as described earlier when the shocks were temporary. It is the presence of cointegration that creates potential problems. Levtchenkova et al (1998) proposes a method for estimating the impact of permanent shocks from models like MSG2. The method proceeds in two stages. In the first stage D(L)e(t) is written as $D(L)H^{-1}He(t) = F(L)He(t)$ where $H=(\alpha_r' \beta')'$ and $\alpha'\alpha_r = 0$. This produces a vector v(t) = He(t) containing *k* permanent and *r* transitory shocks. To see why this is so note that Engle and Granger (1987) showed that $D(1) = \beta_r(\alpha_r'A^*(1) \beta_r)^{-1}\alpha_r$ and $\beta'\beta_r = 0$. Since $H^{-1} = [\beta_r (\alpha'_r \beta_r)^{-1} \ \alpha(\beta'\alpha)^{-1}]$ it follows that the last *r* columns of F(1) = $D(1)H^{-1}$ are zero. This decomposition is due to Granger and Gonzalo (1995). During the second stage the permanent shocks in v(t) are combined together to produce the *k* permanent shocks of the hybrid model i.e. $\varepsilon^p(t) = P^{-1}v^p(t)$, where the "p" designates a permanent shock. After partitioning C(1) and F(1) so that $C_p(1)$ and $F_p(1)$ are the columns of C(1) and F(1) corresponding to the permanent shocks, one has the relation $C_p(1) = F_p(1)P$, and so

$$P = [F_p(1)'F_p(1)]^{-1} F_p(1)'C_p(1). \quad (14)$$

The responses to the permanent shocks are then be computed using $C_p(L) = F_p(L)P$. [6]

Suppose that one designated the cointegrating vectors estimated from the VECM in (8) – using (say) the Johansen (1988) estimator – as b. One difficulty in constructing the hybrid model's permanent shocks is that C(1) is found from simulations of the MSG2 model and not constructed from the VECM. Although one can find a γ such that $\gamma'C(1)=0$, there seems no reason to believe that γ will equal b, *unless it has been assumed in estimation that* $\beta = \gamma$. When the latter is done restrictions on the VECM are produced which can be tested. However, if these are rejected, a question is thereby raised about the utility of constructing an MSG2-type VAR. Essentially, the problem arises because one can no longer really work with the unrestricted VAR in (1). Certainly equation (1) can be estimated (replacing y(t) with x(t)) and used to construct a G(1) by inverting the estimated A(1). However, the estimated G(1) will not have any columns that are set to zero. In contrast, because of the assumption that only *k* of the shocks are permanent, it must be the case that *r* of the columns of C(1) are equal to zero, since these correspond to the transitory shocks, and this makes C(1) singular. The upshot of this dichotomy is that it will be impossible to find a non-singular P that would transform C(1) into the estimated G(1). Applications with permanent shocks that do allow for cointegration tend to follow the strategy in King et al (1991). In that paper it is *assumed* that C(1) has a form such that b′C(1)=0 i.e. C(1) does not come from any fully articulated model but is *designed* so that the long run impacts of permanent shocks are such as to obey the cointegrating relations estimated from the data. This is a useful methodology, but is one that might be hard to generalize.

Some Models of Monetary Actions

Six monthly variables are selected for the empirical analysis: *ly* (log of industrial production), *lp* (log of the CPI), *lpp* (log of a producer price index), *lnbr* (log of non-borrowed reserves), *ff* (the federal funds rate) and *ltwi* (log of a trade weighted exchange rate), measured over the period 1974M1-1996M8. These are the same variables as in the CP model described earlier except that we use producer prices instead of commodity prices and the exchange rate instead of total reserves. The MSG2 model has prices corresponding to producer prices in its simulations and we decided it would be much harder to map these into an index of commodity prices. We also decided that the move to flexible exchange rates, and the increasing exposure of the US economy to international factors, has made it more likely that exchange rates play a part in the transmission mechanism. In smaller countries such as Australia and New Zealand the exchange rate is frequently the primary mechanism whereby monetary actions have an effect.

Pure Transitory Shock Models

As a benchmark we estimate a recursive VAR model, with the variables ordered as described. This is termed the ERVAR model. The VAR includes six lags, a constant and a time trend. The estimation period is obviously chosen to reflect the flexible exchange rate period while the time trend is in accord with the fact that models such as MSG2 describe departures from a steady state growth path. Figures 4-7 show the impulse responses to a 1% monetary shock of the ERVAR model.

As we foreshadowed earlier a hybrid model is available by imposing MSG2 responses upon a VAR. Since the estimated VAR is based on a foundation of transient departures from a constant growth path, we first seek to impose information upon the VAR that reflects the effects of transient shocks within MSG2. Since there are six variables in the system there will be six transient shocks. Accordingly, six experiments were performed on MSG2 in order to generate the information that would be used. These were a temporary (unanticipated) 1% rise in nominal money balances (a monetary shock), total factor productivity (a supply shock), US income taxes, government expenditure in the remainder of the OECD, lending to developing countries, and a 100% rise in commodity prices (an oil price shock).

One might begin with the polar case where the information from all six MSG2 transient shocks is used to find T of section 2. The estimate of that matrix will be $(\Sigma_{j=1,M} G_j)^{-1} (\Sigma_{j=1,M} C_j)$. Figures 4-7 then show the impulse responses to a 1% rise in the money supply when M is set at eighteen months i.e. the hybrid model is made to agree with MSG2 in that its cumulated impulse responses to that point are the same as in the MSG2 model. The hybrid model is termed MSG2VAR. Also plotted in these figures are the impulse responses from the ERVAR and MSG2 models; the latter are just points at each twelve month interval. By design it is impossible to distinguish statistically between the ERVAR and MSG2VAR models.

Figures 4 to 7 about here

The figures are interesting in that they show that the MSG2 information has quite a powerful effect in determining the shape of the impulse responses, but at the same time the data plays a part, particularly over the first year or so. After that point MSG2VAR replicates the MSG2 responses pretty well. It must be admitted that the way in which the impulse responses in MSG2VAR deviate from MSG2 in the short run are not necessarily desirable. In particular, the output response is disappointing. To some extent one can gain an appreciation of why this is occurring by combining the ER model responses with the fact that the cumulative responses in the MSG2VAR model must agree with those of MSG2 after eighteen months. Accordingly, if the large positive movements in output seen in ER after the first year are regarded as describing what is in the data, this effect has to be counterbalanced by some large (or sustained) negative shocks in the first year in order to satisfy the adding up condition. Probably the most dramatic difference between the ERVAR and MSG2VAR models lies in the very large exchange rate response to a monetary shock in the former. Because uncovered interest parity is imposed within MSG2 (over the year) the interest rate and exchange rate responses are virtually identical whereas the ERVAR model produces a much greater exchange rate response.

Given that the shocks ε(t) can be recovered from the VAR residuals e(t) through $T^{-1}e(t)$ it is possible to examine their correlation. Unlike in a recursive VAR, these shocks might not be uncorrelated, as that constraint was not needed to find T. Indeed, to replicate the data it seems likely that they would need to be correlated. The correlation amongst some of the hybrid model's shocks turns out to be close to unity - specifically those relating to US income taxes, rest of OECD fiscal expansion and lending to developing countries. This does not mean that they are identical, as they possess different impulse responses, but that failing to make them correlated would mean one could not reproduce the data as represented by A(L) and V.

The fact that the three shocks just mentioned are so highly correlated suggests that we may not have adequately captured the transient shocks that are in the data. Consequently, it might be desirable to let the data determine some of the shocks. To this end we chose three shocks from the six- money, productivity and oil prices - and then tried to estimate T as explained in section 2. This requires an assumption that all the transient shocks are uncorrelated. However, although we have the same number of equations as unknowns, we were unable to find an exact numerical solution to them. Thus the resulting estimated shocks had some correlation. One possibility is that our equation solving routines were not powerful enough. Another is that the problem arises because the zero correlation restriction is quite incorrect. Some evidence that this might be so is found from the fact that oil and money shocks have a correlation of –0.8 when T is determined using all six shocks. So forcing the correlation to be zero may mean that one cannot find solutions to the equations that would not violate conditions such as non-negativity for the variances.

Combined Permanent and Transitory Shock Models

The six variables that are being modeled are frequently regarded as either being I(1) or very close to it.[7] If the variables are taken to be I(1) there must be permanent shocks impinging upon the system, so that treating them as transitory would ignore important information about the data. Accordingly, it is of interest to construct models that allow for the possibility of permanent shocks. A first question that needs to be answered is how many permanent shocks there are, or, equivalently, how many cointegrating vectors are there?

Table 1 below provides Johansen's (1988) maximal eigenvalue and trace tests for this question, from which the evidence seems to favor r=2. The fitted VAR features 6 lags and a constant term.

Table 1: Johansen's Tests for Cointegration

Maximal Eigenvalue Test

Hypothesis	Test Stat.	Crit. Value (0.05)
r=1 vs. r=2	34.8	33.6
r=2 vs. r=3	17.6	27.4

Trace Test

Hypothesis	Test Stat.	Crit. Value (0.05)
r=1 vs. $r\geq 2$	82.3	70.5
r=2 vs. $r\geq 3$	47.5	48.9

Just as for the transitory shock experiments a benchmark model is needed, and to this end the ERVAR model was modified by assuming that the data could be summarized by the VECM found with Johansen's estimator (after setting r=2). However, the resulting VAR is still assumed to be a recursive one. We term this model ERECM. The determination of the unknown elements of C_0 is done just as for the transitory shock case, but now it is the covariance matrix of the VECM residuals that is subject to a Cholesky factorization. Performing shock identification in this way means that there is no specific separation of shocks into permanent and transitory and it is likely that all the shocks will have permanent effects.

To construct a hybrid model one needs to make some assumption about the identity of the permanent shocks in order to generate usable information from MSG2. Money and labor augmenting technical change seem natural candidates and oil prices have sometimes been suggested as having permanent components e.g. Daniel (1997). This leaves us with the task of selecting one of the remaining shocks used in the transitory analysis to be permanent. Eventually, we decided to choose lending to developing countries. The information to be used then is the long run response of the six variables of the ERVAR model to the four permanent shocks, i.e. $C_p(1)$. In terms of the notation of section 2 these are the first $n–r$ = 4 columns of Ψ_∞. Since the ERECM model identifies shocks by maintaining that the equivalent of Ψ_0 is triangular, it is clear that the two different models source their identifying information from different ends of the $\Psi(L)$ polynomial. Table 2 shows the values of $C_p(1)$ coming from MSG2 with the four shocks just mentioned.

Table 2: $C_p(1)$ Matrix for Four Permanent Shocks

Var\shock	Money	Tech Prog	Oil	LDC Lending
ly	0	0.819	–1.159	0.115
lp	1	–0.800	3.624	–0.215
lpp	1	–0.819	2.828	–0.191
lnbr	1	0	0	0
ff	0	0	0.322	–0.147
ltwi	–0.807	0.403	–5.994	0

One interesting observation that can be made from Table 2 is that the nominal exchange rate does not depreciate by 1% in response to a sustained 1% rise in money balances that eventually increases the US price level by 1%. The reason for this is that the MSG2 model assumes that there are countries that link their currency to the US dollar, so that there is no movement in the bilateral exchange rates between these countries and the US.

Another way to think about the information in Table 2 is to determine what the cointegration vectors underlying $C_p(1)$ are, i.e. the β which are consistent with $\beta' C_p(1)=0$. After normalizing upon *lnbr* and *ltwi* these turn out to be

$$lrnbr = 1.044ly + 2.97(lpp{-}lp) - 0.16ff \tag{15}$$

$$lrtwi = -0.12ly + 4.87(lpp{-}lp) + 0.20lp + 1.83ff, \tag{16}$$

where *lrnrb* and *lrtwi* are real variables, having been deflated by the CPI. The levels variable *lp* enters the real *twi* equation because of the presence of some fixed bilateral rates. Of course cointegrating vectors are hard to interpret, as they need not represent any structural relations. Nevertheless, the first of these does look like a money demand relation. MSG2 does have a money demand function as one of its structural equations except that the scale variable involves real gross output and the price level is for domestic goods.

A test of the cointegrating vectors implied by MSG2 as a restriction on the VECM is strongly rejected and there is an evident trend in the second cointegrating error. To try and understand this phenomenon one could compare the MSG2 relations with those found from the VECM using Johansen's estimator (setting r=2), producing the estimates given in (17) and (18).

$$lrnbr = -5.47ly - 0.78lpp + 3.40lp - 12.10ff \tag{17}$$

$$lrtwi = 2.12ly + 1.30lpp - 1.49lp + 7.92ff. \tag{18}$$

Unfortunately, these do not shed much light upon why the MSG2 cointegrating vectors are rejected by the data. There are, of course, a standard set of suspects such as whether the VAR omits variables that are important and the fact that it might not be stable over time. There are also some specific potential causes. Foremost among these would be that the money stock defined within MSG2 is much broader than non-borrowed reserves. In this connection one might observe that the behavior of that variable since the introduction of "sweep accounts" in 1994 makes it a dubious candidate as the predictor of price level movements. The estimated cointegrating vectors in (17) and (18) point to possible difficulties along those lines. It is also possible that the permanent shocks we have chosen are not the correct ones for the period concerned. However, although some experimentation was done with alternatives, in all cases the implied cointegrating vectors were rejected.

We proceeded to form a VECM using the MSG2 cointegrating vectors and then imposed the long run restrictions implied by the $C_p(1)$ values so as to get a hybrid model, termed MSG2ECM. As detailed in section 2 this involves estimating the matrix P relating the putative hybrid model shocks to the VECM errors. Compared to the transitory shock case, the permanent shocks extracted in this way feature much weaker correlations; the highest being –0.62.[8] Figures 8-11 give the impulse responses to a 1% permanent increase in the money supply for the ERECM, MSG2ECM and MSG2 models.[9] For interest rates and exchange rates the hybrid model tends to exhibit much stronger effects from a monetary shock than MSG2 does, with the opposite being true for output and the price level. Compared to ERECM the responses of the hybrid model are either larger or comparable (in the case of the interest rate). It is interesting that the perverse initial output responses seen with ERECM disappear with the hybrid model. The ERECM does reasonably well in producing long-run output and interest rate responses that agree with MSG2, but deviates by large amounts for the price level and the exchange rate. Increasing the money stock in ERECM by 1% only increases the price level by around 0.38%. Interestingly enough, the ratio of the long run exchange rate to price level changes in the ERECM is quite close to the –0.8 predicted by MSG2. The much debated liquidity effects of monetary policy are displayed in Figure 10. MSG2 predicts that these are smaller than one typically finds from VAR studies which use non-borrowed reserves as the monetary variable (the ERECM and MSG2ECM results are quite representative in this regard) and the effect seems to disappear very quickly in MSG2ECM compared with either MSG2 or the ERECM.

Figures 8 to 11 about here

Conclusion

There has been an increasing use of calibrated theoretical models in economics, particularly for the evaluation of policy options. For long-term responses such models are very attractive, but they have difficulty in matching the short-term dynamics of an economy. Consequently, it is interesting to explore some methods whereby data-based short-term dynamic behavior can be grafted on to these models. This paper looked at how that might be done. The basic strategy was to impose the impulse responses of the theoretical model associated with various policy experiments upon a VAR. The distinguishing characteristic of our investigation has been a focus upon a particular model, MSG2, as the source of the identifying information. MSG2 is an appealing choice as it has been used in a number of policy-making bodies around the world. However, there is nothing in what we do which is specific to MSG2. Our study has thrown up a lot of issues about the possibilities of imposing theory models upon a VAR, and we have not been able to resolve many of these. The problem that the paper addresses seems to be an important one and we hope that our paper will stimulate more work on the topic.

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[1] Complete documentation of the MSG2 model including the theoretical derivation and some evaluation of the tracking performance of the model can be found in McKibbin and Sachs (1991). The specification of the model used in this study (version 43I) can be found in McKibbin (1997).

[2] This model might be regarded as representative of the approach by authors such as Christiano, Eichenbaum and Evans (1996).

[3] In MSG2 the money stock only changes in response to a monetary shocks, and for a transitory money supply shock the money stock immediately returns to its base level, making the total cumulated change just 1%. In the CP model however the dynamics are such that the cumulated change in the money stock is finally 4.67%. One adjustment to allow for this effect is to scale the impulse responses of the latter by 4.67, which produces the numbers in the text. The induced changes in money in the CP model might either arise from the monetary authorities following a feedback rule or simply from the observation that a change in monetary policies rarely tends to be once-off e.g. changes in the Federal Funds rate tend to signal a sequence of changes. If such responses are the cause of the differences a better basis for a comparison might be simulations with MSG2 in which policy is set optimally to achieve some objectives.

[4] In the "just identified VAR" literature attention usually focuses on C_0 (=T). For example Sims (1980) assumes that C_0 is a lower triangular matrix, and hence $n(n-1)/2$ elements of T are known to be zero. In order to determine the other $n(n+1)/2$ elements of T Sims assumes that $\text{cov}(\varepsilon(t)) = I$ so that T may be chosen as the lower triangular Choleski

factorization matrix such that $T^{-1}V(T')^{-1} = I$. This renders (5) a recursive system with the shocks forced to be uncorrelated.

[5] It is also possible that some of the long-run responses to permanent shocks might be taken as unknown rather than being identical to those given by MSG2.

[6] If this H is singular other non-singular choices for H can be used as explained in Levtchenkova et al (1998).

[7] The ADF statistics (with lag lengths chosen with the BIC and with an intercept and trend fitted) are –3.26 (*ly*), –0.79 (*lp*), –3.00 (*lpp*), –1.06 (*lnbr*), –2.38 (*ff*) and –1.79 (*ltwi*). The 0.05 critical value is –3.42.

[8] Because of the way the permanent shocks are formed it is also the case that they are potentially correlated with the transitory ones. It is of course possible to force this correlation to be zero and thereby produce different responses.

[9] Since the only shock specifically identified in the ERECM is a monetary one we only present impulses for that. The impulse responses for productivity shocks from the MSG2 and hybrid models were similar for output, prices but radically diverged for the exchange rate. Even after twenty years they showed different effects. At that point the hybrid model predicts an appreciation of the exchange rate of around 0.4%, which is the long-run change according to MSG2. However, after the same period, MSG2 signals a depreciation of around –0.2%. Thus the MSG2ECM responses for this variable converge to the long-run solution much faster than does MSG2 itself.

Figure 1: Response of Output to Transitory Money Shock.
CP Model (——) and MSG2 Model (□)

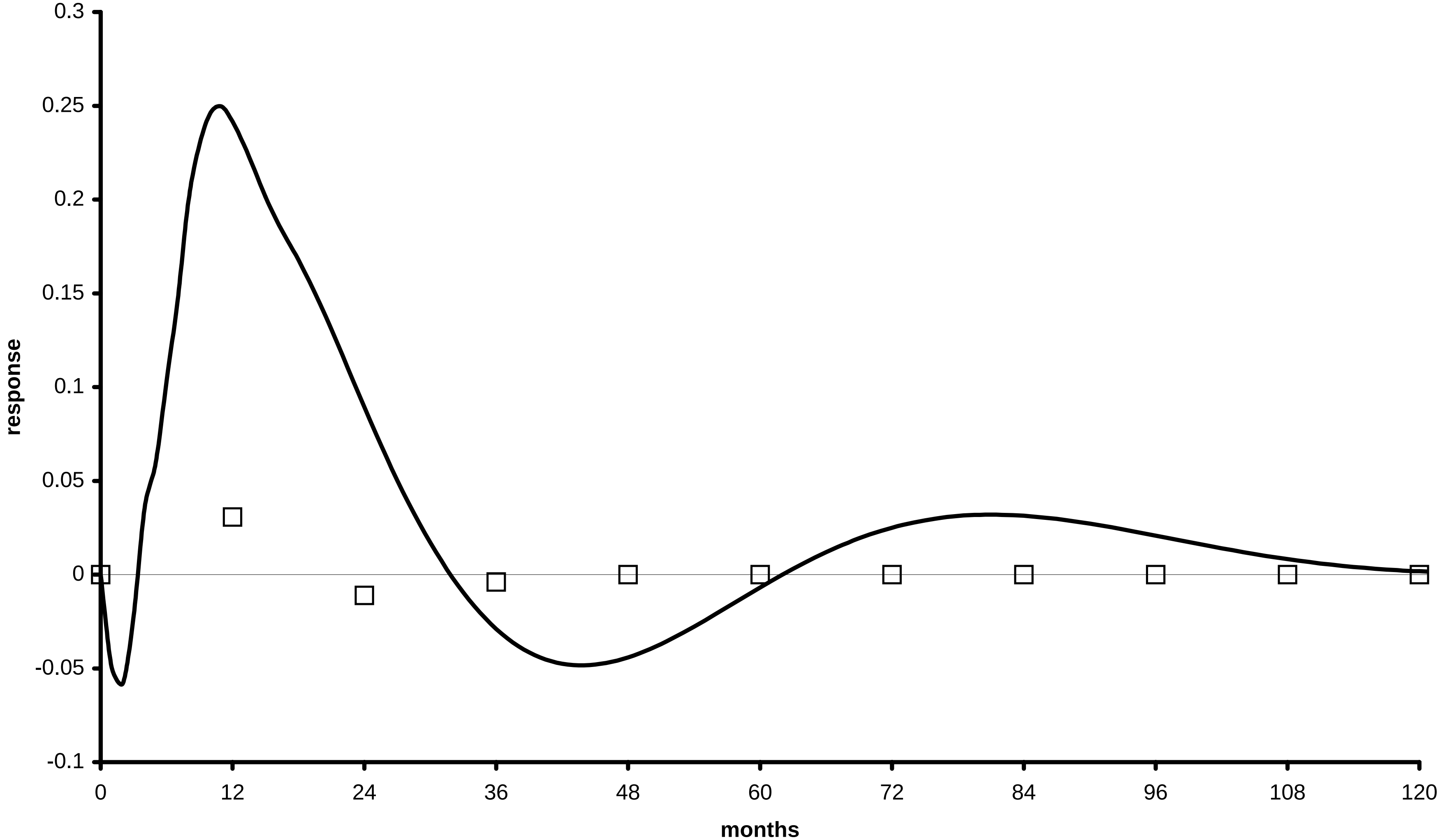

Figure 2: Response of Price to Transitory Money Shock.
CP Model (——) and MSG2 Model (□)

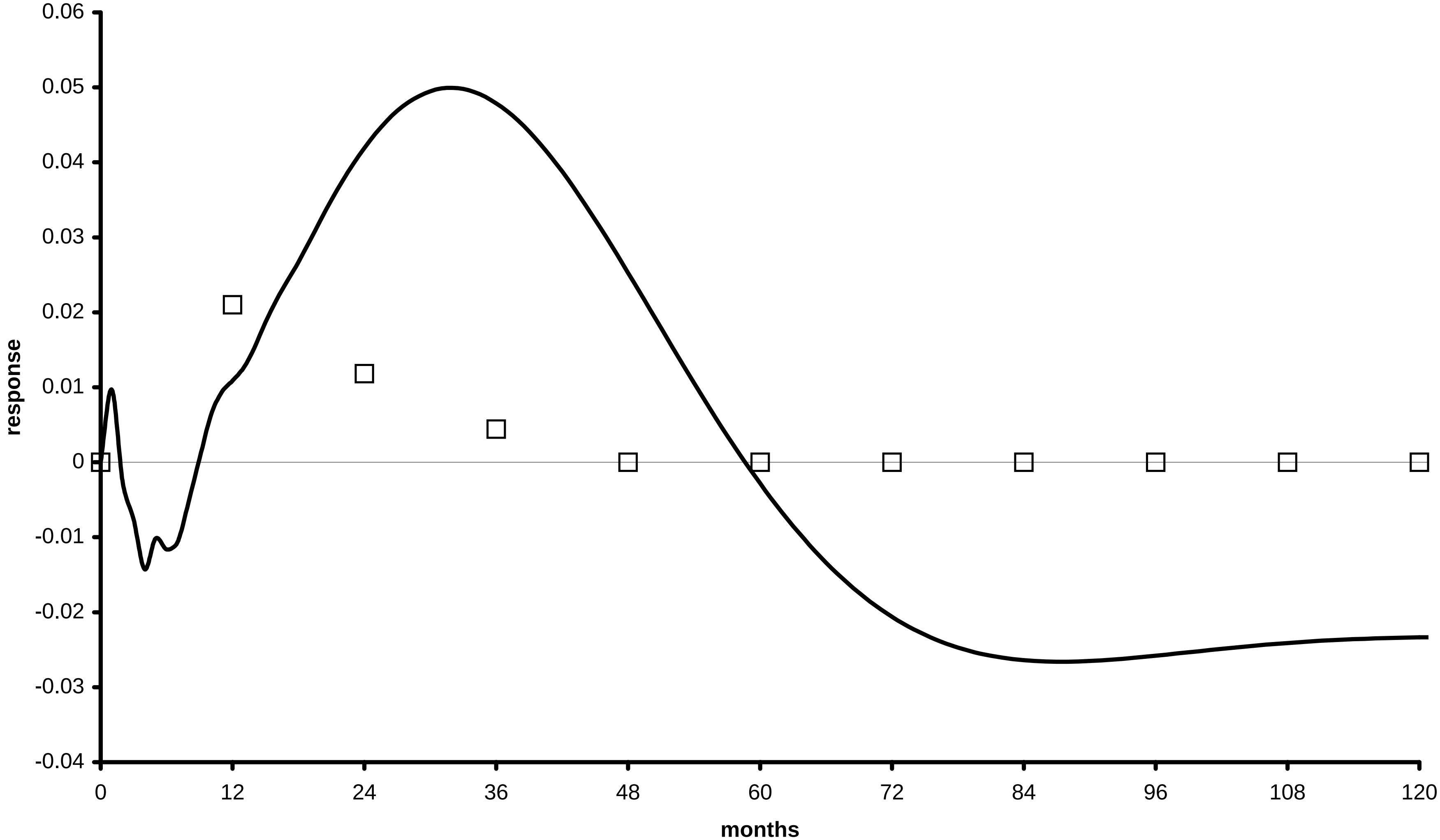

Figure 3: Response of Short-term Interest Rate to Transitory Money Shock.
CP Model (▬) and MSG2 Model (□)

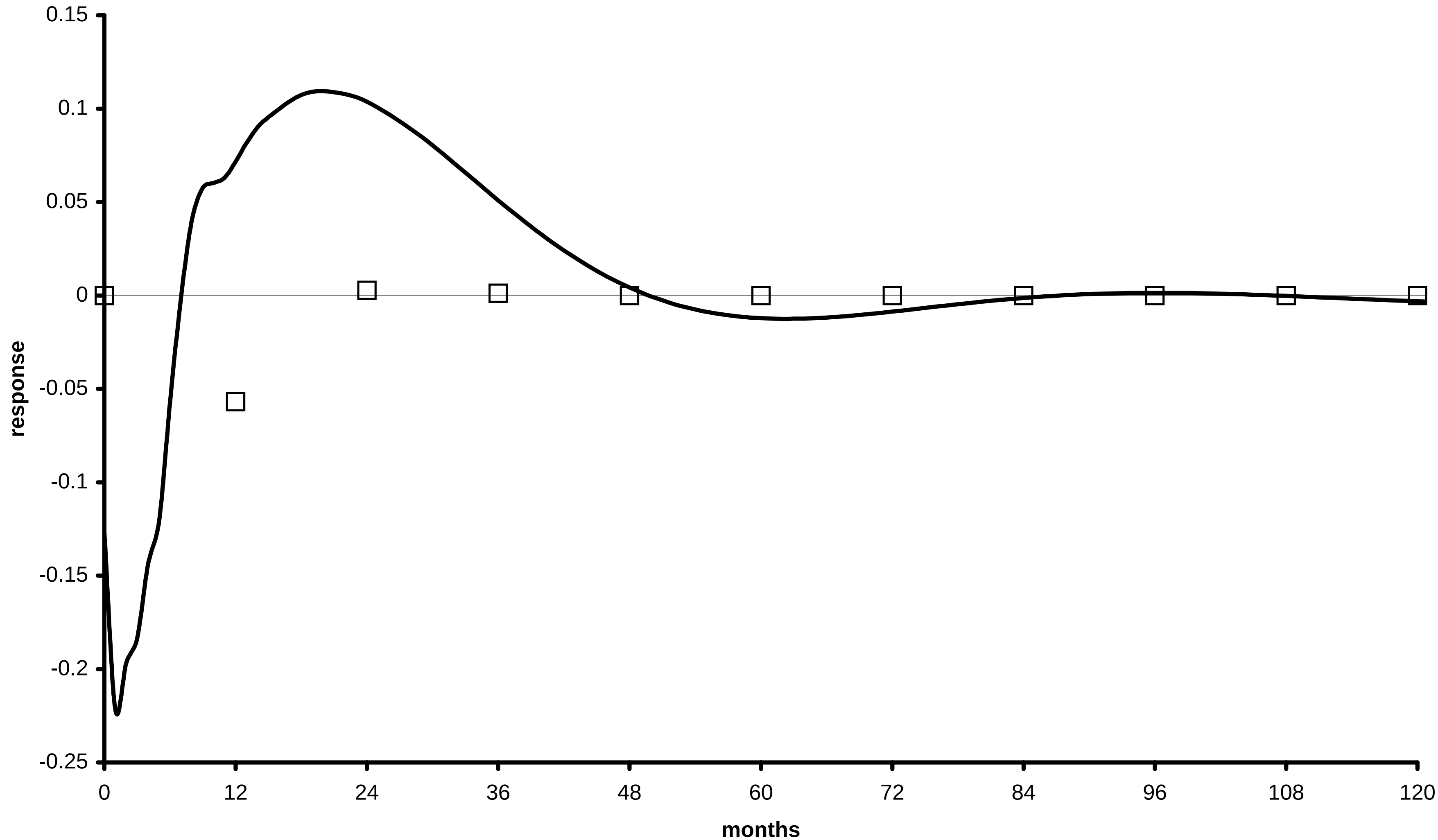

Figure 4: Response of Output to Transitory Money Shock.
MSG2VAR Model (▬▬) ERVAR Model (——) and MSG2 Model (□)

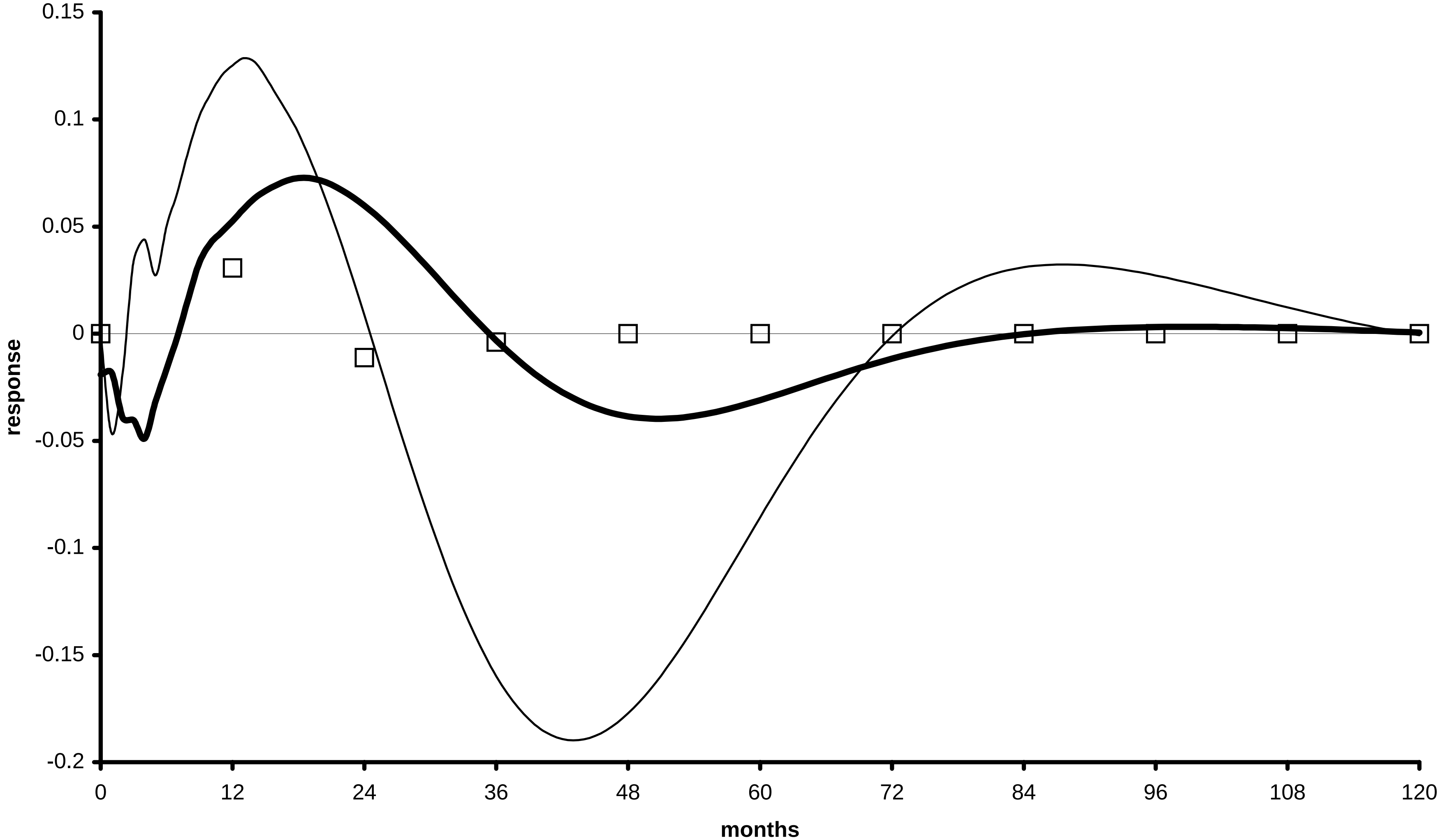

Figure 5: Response of Price to Transitory Money Shock.
MSG2VAR Model (▬) ERVAR Model (—) and MSG2 Model (□)

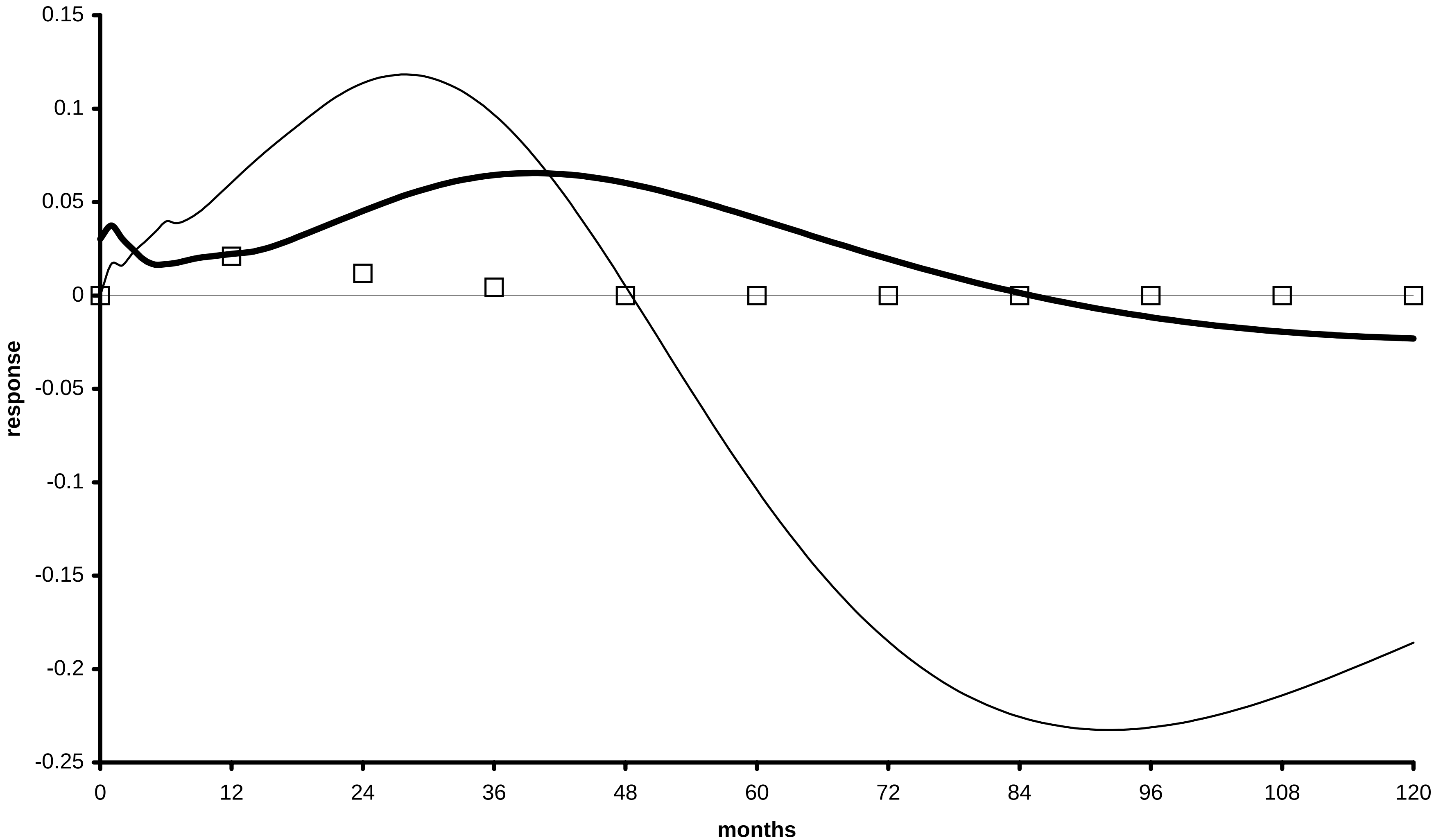

Figure 6: Response of Short-run Interest Rate to Transitory Money Shock.
MSG2VAR Model (▬▬) ERVAR Model (——) and MSG2 Model (□)

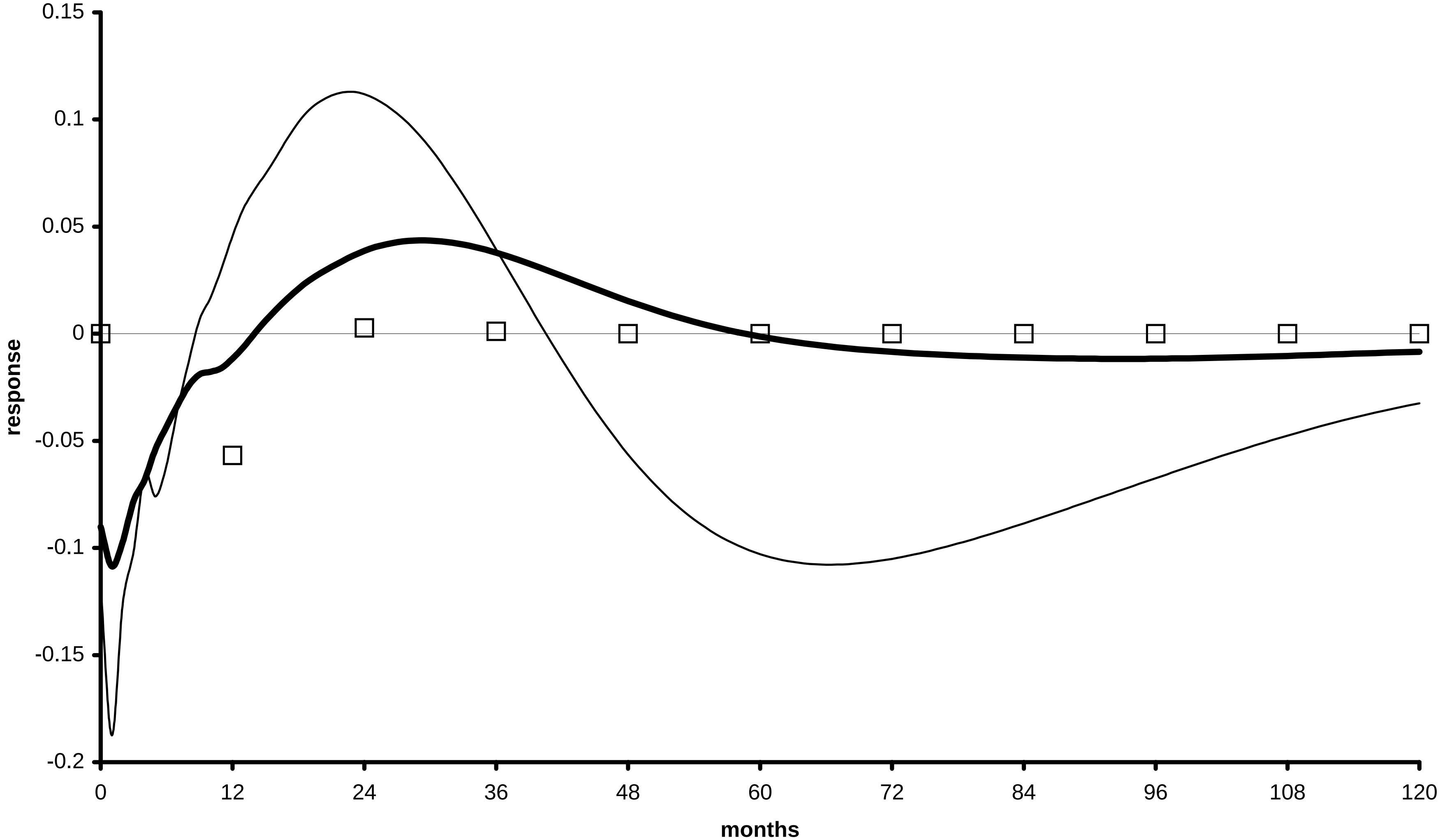

Figure 7: Response of Exchange Rate to Transitory Money Shock.
MSG2VAR Model (▬) ERECM Model (—) and MSG2 Model (□)

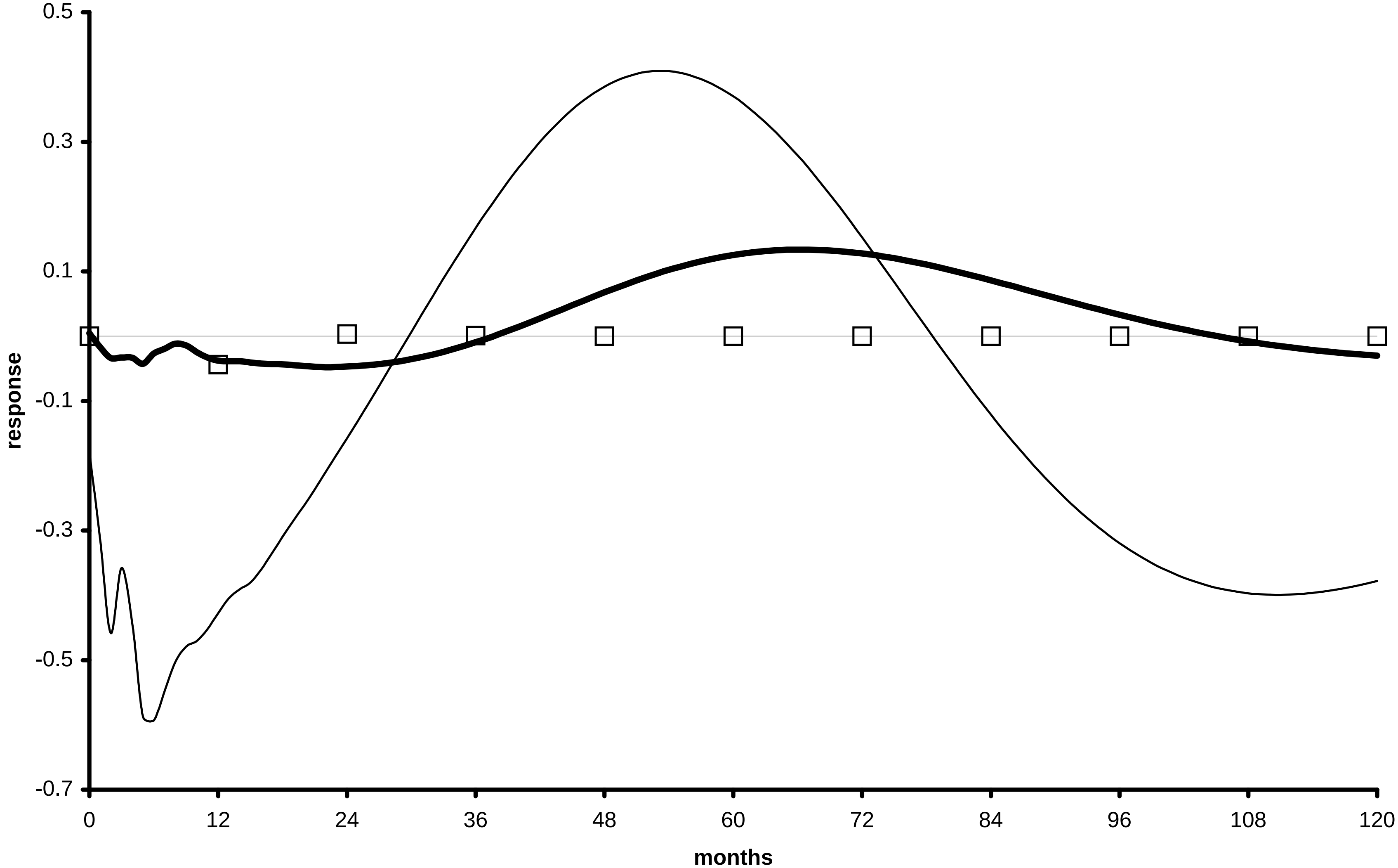

Figure 8: Response of Output to Permanent Money Shock.
MSG2ECM Model (▬▬) ERECM Model (——) and MSG2 Model (□)

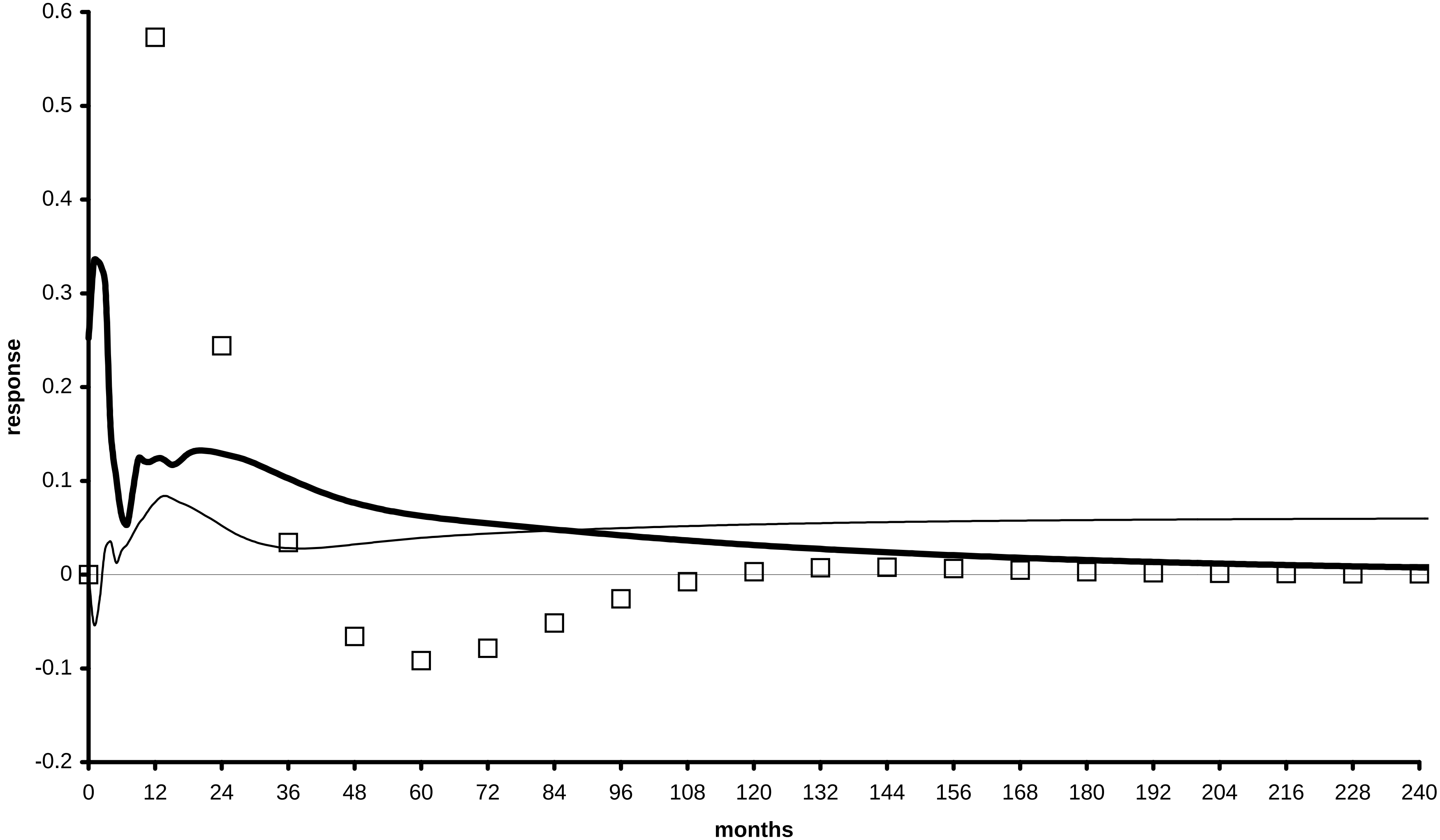

Figure 9: Response of Price to Permanent Money Shock.
MSG2ECM Model (▬▬) ERECM Model (——) and MSG2 Model (□)

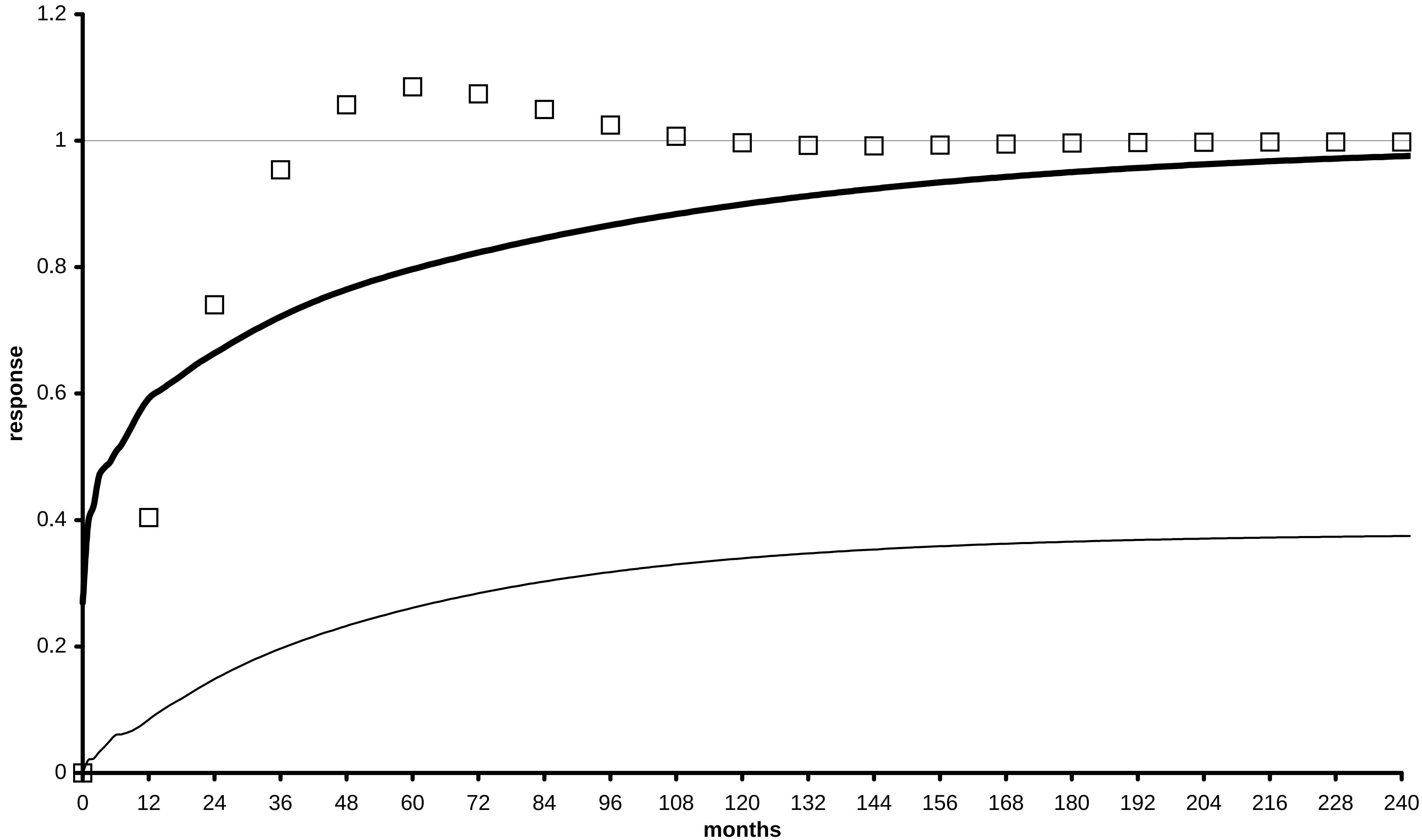

Figure 10: Response of Short-term Interest Rate to Permanent Money Shock.
MSG2ECM Model (▬▬) ERECM Model (——) and MSG2 Model (□)

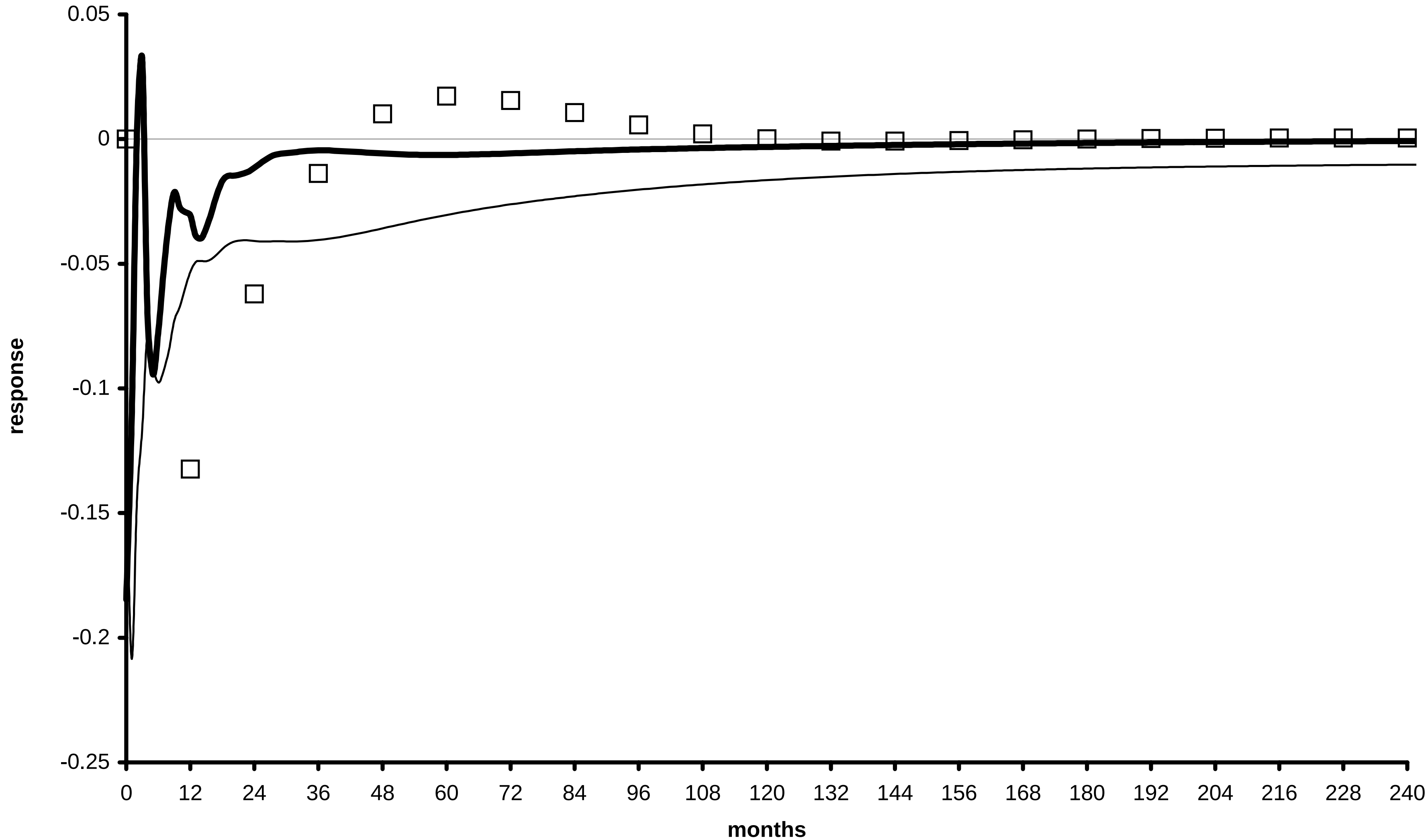

Figure 11: Response of Nominal Exchange Rate to Permanent Money Shock.
MSG2ECM Model (▬▬) ERECM Model (——) and MSG2 Model (□)

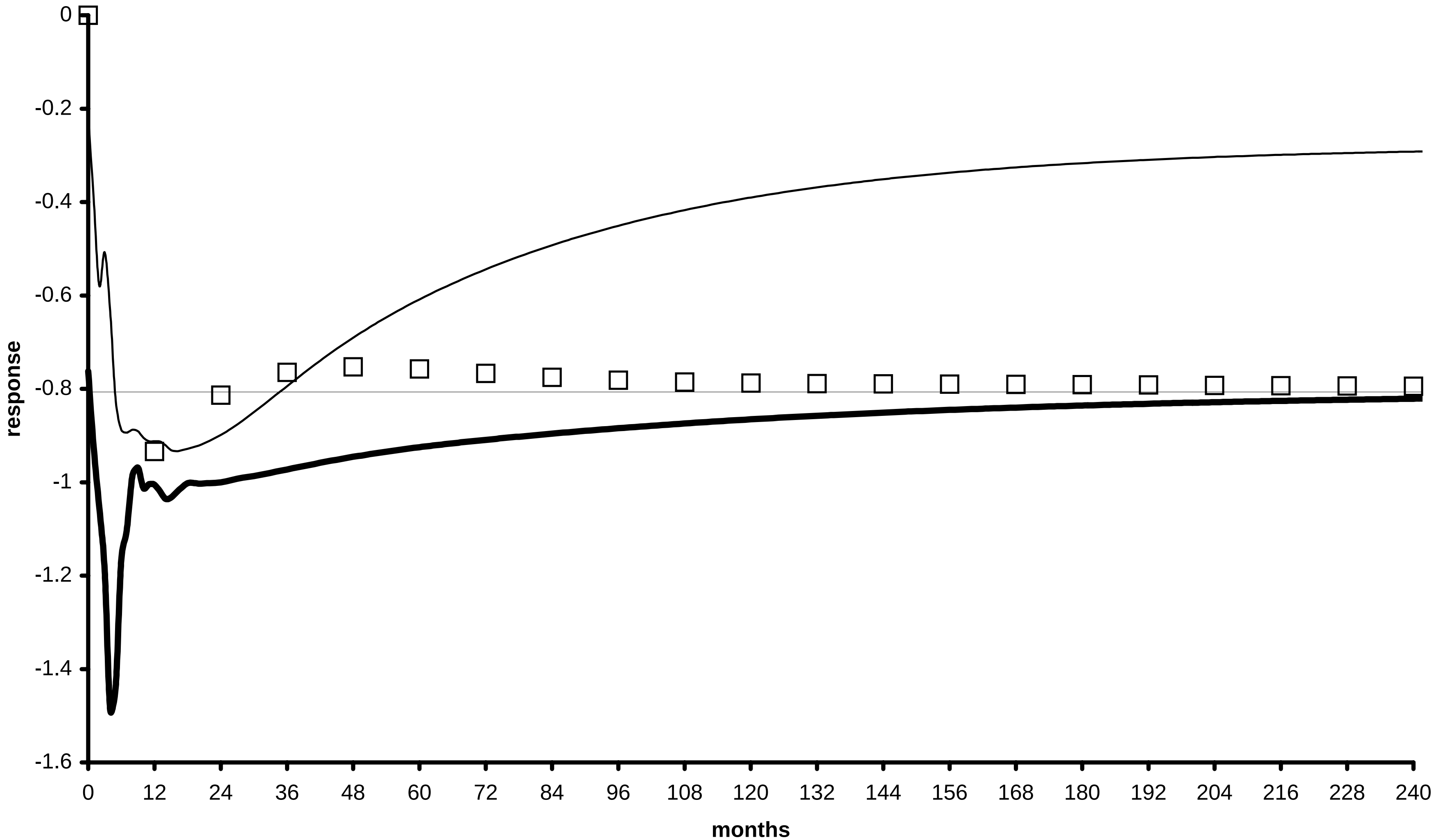

Figure 12: Response of Output to Permanent Productivity Shock.
MSG2VAR Model (▬) and MSG2 Model (□)